

Applied Game Theory And Strategic Behavior Chapter 1 and Chapter 2 review

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Introduction

The book „Applied Game Theory And Strategic Behavior“ is written by Ilhan Kubilay Geėkil and Patrick L. Anderson. The following text is a short review about Chapter 1 and Chapter 2 of book „Applied Game Theory And Strategic Behavior“.

Based on similarity some examples is changed to less abstract and more real examples. Also lot of material is taken directly from that book. The overall concept is same as in the book. Same headings were used and same subheadings also were used.

This overview based more practical examples and less theory. So all definition is given as short as possible. Also in chapter 1 the history part events are describe as short as possible. And chapter 2 the Strategy and Game Theory Concepts part gives overview of definitions in a first place and then some examples are given aswell.

Preface

You need to learn the rules of the game. And then you have to play better than anyone else.

Albert Einstein

Purpose of this Book

The purpose of this book is to demonstrate the use of game theory techniques to address practical issues in business, applied economics, and public policy; as well as to demonstrate the benefits of strategic thinking that incorporates uncertainty about the behavior of other parties.

Instead of providing pure theory, this book concentrates on the practical applications of theory. Applications we present in this book will show how to build a model in an interactive decision-making environment in order to analyze real-world problems.

Organization of Book

1. We introduce a brief history of game theory in [Chapter 1](#). Readers will find the development of the field and introduction of key game theory concepts and inventions.
2. If a reader does not have any interest in the history of game theory, we recommend jumping to [Chapter 2](#), where we introduce game theory concepts and strategy. In that chapter, we show how to illustrate a game, introduce the rules of the game, key concepts, as well as strategy and strategic behavior in game theory.

Chapter 1 A Brief History of Game Theory

Why Study Games?

The field known as “game theory” was invented in the last century by mathematicians and economists as a tool to analyze both economic competition and political conflicts. The fundamental insight of game theory was to apply the logic of games—

in which players compete against each other using strategy, tactics, and effort—to events in real life.

Rapid Discoveries in the Twentieth Century

The theorists began studying game theory about a half century ago.

Key Conceptual Developments in Early Years

1. In 1928, The *extensive (or tree) form* of a game, introduced by John Von Neumann.
2. The definitions of a strategy, strategic behavior, and strategic actions.
3. The normal form (or matrix form) of a game.
4. The formulation of a game into a matrix structure.
5. The concept of mixed or randomized strategy.
6. The concept of individual rationality.
7. Different types of information concepts, such as perfect information.
8. The mini-max theorem.

Pioneers of Game Theory and Advancement

1. In 1944, John Von Neumann and Oskar Morgenstern published the *Theory of Games and Economic Behavior*.
2. During the 1950s, John Nash theory tools and concepts for general *non-cooperative theory* and *cooperative bargaining theory*.
3. In 1950, *Strategy in Poker, Business and War* by John McDonald was published.
4. In 1951 “Nash equilibrium” of a strategic game in.
5. The first textbook on game theory, *Introduction to the Theory of Games*, was published in 1952 by John Charles C. McKinsey.
6. In 1952 the University of Michigan along with the Ford Foundation sponsored “Design of Experiments in Decision Processes.”
7. Savage principle in 1954
8. Lloyd Shapley brought *conditional games* to game theory and defined the specifics of the conditional games.
9. D.B. Gillies and John Milnor developed the first *continuous* game theoretical models.
10. Harold Kuhn contributed to the field with his work on behavior strategies.
11. Melvin Dresher and Merrill Flood of the RAND Corporation developed the structure of the prisoner’s dilemma in Santa Monica in the 1950s.
12. In 1955, it was applied to philosophy by British philosopher R. B. Braithwaite in his book *Theory of Games as a Tool for the Moral Philosopher*.
13. In the late 1950s Kuhn, Kissinger, and Schelling contributed to the field while developing cold war strategies.
14. In the In late 1950s is the use of the *Folk Theorem* to show the solid relationship between repeated and cooperative games.
15. In 1957, *Games and Decisions*, by R. Luce and H. Raiffa, was published.
16. John C. Harsanyi developed games with incomplete and asymmetric information.
17. In 1960, *The Strategy of Conflict* by Thomas Schelling was published. Schelling introduced the *focal point* concept, also known as the Schelling point.
18. In 1966 Harsanyi introduced cooperative games, in which commitments, contracts, agreements, threats, and promises are enforceable.

19. In 1969, D.K. Lewis formalized the *common knowledge* assumption.
20. In 1972, the *International Journal of Game Theory* was first published. It was founded by Oskar Morgenstern.
21. In 1973, Harsanyi introduced the idea of explicit randomization in game theory.
22. In 1974, Robert Aumann introduced the *correlated equilibrium*.
23. In the 1970s the game theory was applied to biology by John Maynard Smith in his work on evolutionary stable strategy (ESS).

Game Theory's Evolution during the Last Three Decades

1. In 1981, Elon Kohlberg published *Some Problems with the Concept of Perfect Equilibria*.
2. In 1982, "Sequential Equilibria" by David Kreps and Robert Wilson was published in *Econometrica*.
3. In 1984, David Pearce and Douglas Berheim independently introduced the idea of rationalizability in their papers *Rationalizable Strategic Behavior* and *the Problem of Perfection*.
4. In 1984, *The Evolution of Cooperation* by Robert Axelrod was published.
5. In 1986, Elon Kohlberg and Jean-Francois Merterns published *On the Strategy Stability of Equilibria*.
6. In 1988, *A General Theory of Equilibrium Selection in Games* by John Harsanyi and Reinhard Selten was published.
7. In 1989, the journal *Games and Economic Behavior* (GEB) was first published.
8. In 1990, *A Course in Microeconomic Theory* by David Kreps was published.
9. In 1994, *Game Theory and the Law* by Douglas Baird, Robert Gertner and Randal Picker was published.

Chapter 2 Strategy and Game Theory Concepts

Thus, what is of supreme importance in war is to attack the enemy's strategy.

Sun Tzu

As an interactive decision-making environment, game theory offers valuable tools for solving strategy problems in everyday life and in the business world. Game theory has been used for determining cold war strategies, establishing merger and acquisition strategies, picking Supreme Court judges, as well as measuring the market power of firms.

Game Theory, Strategy, and Strategic Behavior

A game-theoretic model is an environment where each decision-maker's actions interact with those of others. In general, behavior that involves such interactive decision-making is called *strategic*, and the set of actions and moves by each player with respect to others, given the rules of the game, is called *strategy*.

More on Strategic Behavior and Strategy

We will consider "strategic" behavior as behavior serving the self-interest of the person, based on the person's own subjective evaluation of likely events and the possible actions of other players; with the potential rewards and risks being considered over single or multiple period(s).

Strategic behavior is more than simply maximizing utility with defined prices and commodities. Consider, in the standard one-period neoclassical economics model, a consumer's choice among different commodities to purchase, given that consumer's preferences and the prices that prevail in the market. This is an economic decision but not a strategic decision. Indeed, it is an economic decision that is the basis for microeconomics in a purely competitive market. However, it is not a strategic decision. Because strategic behavior involves considering the *future consequences* of the actions of *more than one person*. Thus, it is different than the standard microeconomic study of consumers and producers.

Game Theory and Strategic Behavior in Business

Strategic behavior occurs regularly among executives, managers and investors in business. In decision-making situations, the person confronts not only uncertainty about future states of nature, but also uncertainty about actions that other persons will take. Using any of the definitions of "strategy" introduced above, this behavior is called "strategic behavior."

The formal structure of game theory models forces each player to consider the actions of others when picking their strategy. The extensive or normal forms of the game lay out the information that each player knows when he is choosing his move. Simply laying out the structure provides significant benefit to a decision-maker.

Given these advantages, game-theoretic models are very powerful tools for analyzing firm decisions. Most of business world scenarios can be modeled as dynamic games with multiple stages, in which one player may respond to the moves of his opponent. In such a world, firms act strategically in each stage, based on the information available at the current time.

Consumer Behavior, Utility Theory, and Game Theory

The important assumptions of game theory is that economic agents are rational players. The goal is to maximize well-being, i.e., utility. Utility within such models have been represented by the payoffs at the end of the game. Utility is an indicator of a person's overall happiness. Consumers make choices to maximize their utility. To model consumers' preferences, we use utility functions. Utility functions are tools for assigning a number to consumption bundles of consumers.

Economists say a bundle (x_1, x_2) is preferred to a bundle (y_1, y_2) if and only if the utility of (x_1, x_2) is larger than the utility of (y_1, y_2) . Symbolically, (x_1, x_2) is preferred to (y_1, y_2) if and only if $u(x_1, x_2) > u(y_1, y_2)$. This assumption is very important; it is called an axiom of utility theory. If consumer prefers X to Y and Y to Z then this can be marked as $U(X) > U(Y) > U(Z)$.

If you multiply these assigned measures by a positive number, you still have the same order. Let us say $U(X) = 3$, $U(Y) = 2$, and $U(Z) = 1$, so $U(X) > U(Y) > U(Z)$. For example, multiplication by 2 does not change the preference order: $U(X') = 3 \times 2 = 6$, $U(Y') = 2 \times 2 = 4$, and $U(Z') = 1 \times 2 = 2$; therefore, $U(X') > U(Y') > U(Z')$.

Cardinal Utility

Utility theories where the magnitude of utility is important are called "cardinal utility theories." Cardinal utility theory asserts that the level, as well as the order, of utility gained from a bundle of goods and services is significant. For example, one prefers a specific bundle at least three times more than another if he is willing to pay three times as much for that bundle.

Choice Behavior and Game Theory

In game theory, *choice behavior* is determining whether one bundle or another will be preferred. In our game theory models, we consider which strategy brings greater utility. The utility levels of preferences, and the knowledge of how much larger the utility is, are significant for the game-theoretic models.

Utility Functions and Game-Theoretic Models

Some game-theoretic models require a *utility function*. A utility function maps the utility of a bundle of goods and services to a real number. Examples of utility functions: a multiplicative function $u(x_1, x_2) = x_1x_2$, additive function $u(x_1, x_2) = ax_1 + bx_2$ and maximum function $u(x_1, x_2) = \max\{ax_1, bx_2\}$. Some point in a game a player needs to pick a strategy from the strategy set $A = \{A_1, A_2\}$. Depending on utility functions and arguments A_1, A_2 player make one's decision.

Utility Theory and Payoffs

In the real world, properly constructing the payoff matrix is a critical, and often difficult, step. This difficulty arises from the fact that a person's utility is rarely defined by any onedimensional measure, such as a price, quantity, or size, of a cash payment.

Game-Theoretic Models and Illustration

Game theory models describe strategic interaction among many players. Players make rational decisions to maximize their expected utility.

The interdependence of the players' decisions is the foundation of game theory. Game theory models often exhibit a social science corollary to a fundamental principle of physics: Every action has a reaction. These interactions arise in two ways: sequential and simultaneous.

Sequential interaction refers to each player taking action in a sequence of turns. During a player's turn, he or she is aware of the actions taken in previous turns. Furthermore, each is aware that his current action(s) will affect later actions of the other player(s), as well as his future action(s) during the game.

The second kind of interaction is simultaneous. Simultaneous interactions occur when players take actions concurrently, in ignorance of the others' current actions. It is important to note that while players do not know the specific actions of the other players, they are aware of each other in simultaneous games.

The Payoff Matrix and Tree Diagram

To analyze sequential-move and simultaneous-move games, we the payoff matrix and the tree diagram can be used. The payoff matrix illustration in game theory is also called the "normal form" of the game.

A tree diagram is tool for illustrating and analyzing games, in which players act sequentially. A common depiction of possible outcomes over time, and one that we will use repeatedly, is called a "tree," "event tree," or "decision tree." Such a device combines the time aspect, and the logical and sequential connections among decision-makers and events.

The tree starts at a specific point in time, which conveniently can be drawn as a single point in a two-dimensional space. From that point, "branches" of the tree emerge. Each such branch terminates

into another “node.” At each “node” of the tree, an event occurs such as a decision, random occurrence, or receipt of information. This event then causes the tree to branch into different outcomes at the next time step. In sequential move games, players observe the other players’ future moves and use them in assessing their best current move. This is known as “backward induction,” that is, *look ahead and reason back*. Tree diagrams, generally used for sequential or the non-simultaneous games, are also called “decision trees,” or the *extensive form* of the game.

Strategic Thinking and Simultaneous- and Sequential-Move Games

In sequential-move games, the current course of action taken by the player is based on ones expectation of what the other players’ future strategy and action will be. A player typically thinks, “if I choose this course of action, the other player will take that course of action, therefore I should do this,” and so forth.

In contrast, simultaneous-move game interaction can be more difficult for players. This is because players must guess what the other player is anticipating at the moment, and respond accordingly, as well as anticipate how these actions affect future outcomes of the game. For simultaneous-move games, a player may try to think of more combinations of his and other players’ actions.

Rules of the Game

The nature of a game theoretical model is determined by the rules. The key rules of any game:

1. *Players*. How many players do we have in a game? Are their interests matching or conflicting?
2. *Information*. What information does each player possess? Do they have *complete*, *symmetric*, or *perfect* information regarding each other’s actions and payoffs? What are the moving sequences of players?
3. *Actions or Strategies*. What actions or strategies are the players allowed to have? What are the specifics of interaction between players? Are they allowed to communicate?
4. *Payoffs*. What are the possible outcomes for each player? What is the utility or expected utility for each player at the end of the game for every action they are allowed to have?

Players

Players are rational economic agents, make decisions with a well-defined set of actions and strategies. Their goals are to maximize their utility or expected utility. *Rational* characteristics for the players in gametheoretic models are implicitly assumed for every game we analyze in this book.

Information

Information is the knowledge each player has about the game. The information set may include the number of players, each player’s set of actions, strategies, payoffs, and the moving sequence. Players in a game possess some sort of information set; they might have *perfect* or *imperfect* information, *complete* or *incomplete* information, *symmetric* or *asymmetric* information.

Important implicit assumption of game theory is that the structure of the game is common knowledge: that is, players know how many players are in the game, their moving sequence, either simultaneous or non-simultaneous, and the set of actions or strategies available to each player when he or she moves.

We have three main categories for the information structure of a game :

1. Perfect vs. imperfect information,
2. Complete vs. incomplete information, and
3. Symmetric vs. asymmetric information.

Perfect vs. Imperfect Information

Perfect information means that no moves are simultaneous, and each player knows the sequence of moves and where players move. All simultaneous-move games are games of imperfect information. An incomplete or asymmetric information game is also a game of imperfect information.

Complete vs. Incomplete Information

In a game of incomplete information, there are some uncertainties about the actions of players, the moving sequence of the game, or the payoffs. A game of incomplete information might include probabilities at some of the nodes in the game. A game of incomplete information is also a game of imperfect information.

Symmetric vs. Asymmetric Information

In a game of symmetric information, players have the same elements in their information sets, including the sequence of the game, where each player chooses an action, and the end nodes. Otherwise, a game is called a “game of asymmetric information.” In asymmetric information games, players have different information regarding each other’s moves or payoffs.

Set of Actions and Strategies

In a game, each player has an action set that includes their possible moves or strategies. Players determine their strategies based on the information available to them at the beginning of the game and at each node.

Payoffs

Payoffs are what players receive at the end of the game. The nature of games is that the payoffs differ depending on the actions of the players. The possible payoffs are visualized in a payoff matrix.

Strategy and Equilibrium

A “strategy” is an order of moves determined in advance of some events by an individual player. A strategy can, and often does, depend on the action of other players, random events, and particular payoffs available to a specific player. In games where there are more than one move for each player, a “strategy” is different than an “action.” In general, players decide what action to take by using a strategy.

Dominant and Dominated Strategies

We can define two characteristics that apply to some strategies in specific games, based on the likely outcome for the player. If any player follows a *dominant strategy*, the player will get the best possible payoff regardless of what the other player(s) will do. A dominant strategy is the *optimal* strategy for a player no matter what the other player(s) does (do).

Dominated Strategies

Just as a dominant strategy is a strategy that is better than any other strategy a player can choose from his set of actions, a dominated strategy is a strategy that is worse than another strategy available for the player. In a game with rational players, players can be expected to play their dominant strategies (if they have any) and avoid their dominated strategies.

Equilibrium

An *equilibrium* in game theory is defined as a stable outcome, based on the payoffs received by players at the end of the game. We call it stable because after players settle on an equilibrium point with their payoffs, they have no incentive to deviate from that point. When we have an equilibrium point in a game, we call that the *solution* of the game.

Dominant Strategy Equilibrium

In parlor games, there is no strategy that is dominant. In some games, players either have dominant strategies, or learn their best strategies over time and begin to play them repeatedly. This creates the potential for an equilibrium position among multiple players.

It is possible that every player in a game has a dominant strategy. It is also possible that no player has a dominant strategy. If there is a dominant strategy for each player, then we have a *dominant strategy equilibrium* for that game. If there is a dominant strategy for only one player, we have a dominant strategy equilibria in a 2-player game. If it is an n-player game, we may or may not have a dominant strategy equilibrium.

Nash Equilibrium

We do not have dominant strategy equilibria in all games. In a two-player simultaneous-move game, we call a pair of strategies a *Nash equilibrium*, if player I's choice is optimal based on player II's choice, and player II's choice is optimal based on player I's choice.

If a game is a non-simultaneous (sequential) game, the first mover has the advantage and is able to dictate an equilibrium. However, in a simultaneous-move game, we do not have such an attribute. Do Nash equilibria exist in every game? The answer is no.

Note on Dominant Strategy Equilibrium and Nash Equilibrium

Even though dominant strategy equilibria are stable, players do not have a dominant strategy in every game. The Nash equilibrium occurs in a broader spectrum of games.

A game has a Nash equilibrium if there exists a set of strategies such that each player optimizes his utility given the other players' actions. A Nash equilibrium is quite stable, because no player has an incentive to deviate from his *Nash strategies*.

Recall that in game theory, we implicitly assume players are rational agents. The Nash equilibrium is dependent upon individual rationality more than the dominant strategy equilibrium. The equilibrium outcomes in the Nash equilibrium depend not only on every player's own rationality as in the dominant strategy equilibrium, but also on other players' rationality.

Subgame Perfect Nash Equilibrium

Subgame is a smaller portion of a game starting at a specific node of an entire game. From that point, the subgame emerges and continues to the end of the whole game. We call an equilibrium a *subgame perfect Nash equilibrium*, if every subgame of the entire game has a Nash equilibrium based on players' strategies.

Mixed Strategies; Repeated Games

“pure strategies”- players making one choice and playing the game with that choice only. If games are played more than once, we call them *repeated* games. In such cases, it is possible for players to change their strategies. Players are even allowed to randomize their strategies.

In the game let us assume that player I picks A 70% of the time and B 30% of the time. This type of strategy is called a *mixed strategy*. If a player has such a mixed strategy, the equilibrium of the game changes. Let us reintroduce our sample game here: We assumed that both players I and II have the information about players I's mixed strategy of choosing A 70% of the time and B 30% of the time. For simplicity, let us assume player II does not randomize his strategies. Player II's *expected payoffs* of the new game becomes 2 for choosing C, $[(2 \times 0.7) + (2 \times 0.3) = 2]$, and 2.4 for choosing D, $[(3 \times 0.7) + (1 \times 0.3) = 2.4]$. The games shown up to this point have always had a Nash equilibrium in mixed strategies.

Maximin Strategy

What strategies are available to players who face payoffs that vary according to their opponent's actions? The maximin strategy pertains to a two-person zero-sum game. If a player (player I) attempts to take action(s) to reduce the other player's (player II) payoff, player II will take the action(s) that will give him the maximum minimum payoff. It should be noted that since most games are not *zero-sum games*, the maximin strategy is often not applicable.

Sequential Games and Problem Solving

Let us consider sequential games. This time we assume that this game is a non-simultaneous-move (sequential) game, and player I moves first. We also assume that both players know the sequence of their moves. In this non-simultaneous game, moving first gives player I a *first-mover advantage*. It is important to note that by changing the sequence of the players' moves, we have a different equilibrium for this game. As we indicated above, rules and information are very important in game theory, as changing the rules will often change the outcome of the game. Let us change the sequence of players, and see what would happen. Note, in this case, that first figure in the payoff vectors belongs to player II. By using backward induction, we have the reduced form of the game. By choosing D at the beginning of the game, player II becomes better off, $[3 > 2]$. Note that when player I moves first, the equilibrium outcome of the game is (B, C); when player II moves first, the equilibrium outcome of the game is (A, D).

Complex Games and Games by Categories

In real life, games are more complex. Simple games to illustrate game theoretic concepts were used; Some examples about the complex game aswell.

n-Person Games

There are games with no certain number of players. We call games with more than two players, n-person games where n can be 3, 4 or more. Economists and mathematicians have used advanced techniques to analyze n-person games and find equilibria for this type of games. In such games, the rules of the game, information set, and actions available are very important, just as in two-person games. In n-person games, *power* is the key element for the outcome of the game. We define power here as the ability to affect the outcome of a situation to one's favor. In a game, this can be seen as who will win the game, given the power each player has to affect future outcomes such as play

during the game, as well as the final outcome, the winner of the game. Power might have different forms in game theory: market power, product power, power due to connections, or power from information. For complex games, a forward looking approach is a very powerful analytical tool. By utilizing a forward looking method, a player can assess what kind of position he will be in after several moves. Some games are complex because of the high number of strategies available to players. A player needs to assess each strategy carefully, as well as his opponent's.

Different Categories of Games

Zero-Sum Games vs. Non-Zero-Sum Games

In zero-sum games, one player's gain is another player's loss. In zero-sum games, the sum of the payoffs of all players should be zero. Zero-sum games are also called *constant-sum games*. Most of the games in real life and the business world are not zero-sum games. In non-zero-sum games, all players could win or lose together. In most of the business partnerships and international trade, we have a *win-win* situation, that is, all parties might gain from the partnership or trade. In zero-sum games, players have no common interests. However, in non-zero-sum games, players have common and conflicting interests.

Static vs. Dynamic Games; Repeated Games

In real life, most of the games are played more than once. Play may unfold differently with a sequential game; that is where the players play the game more than once consecutively. This type of game is called *dynamic*. Repeated games are dynamic. An example of a real life dynamic game would be one where firms set prices periodically. In dynamic games, unlike static games, players observe other players' behaviors, modify their strategies accordingly, and develop reputations about their own behavior.

Cooperative vs. Non-Cooperative Games

A game can be cooperative or non-cooperative by nature. This categorization is important for understanding the behaviors, objectives, and strategies of the players. A game in which players are allowed to cooperate with each other on a joint strategy is called a "cooperative game." In cooperative games, agreements, commitments and threats are binding and enforceable. An example of a cooperative game is a bargaining game between parties in a transaction (some sort of merger or acquisition) over the value of a target company. A game in which players are not allowed to cooperate or negotiate on a contract is called a "non-cooperative game." In these games, commitments are not enforceable.

Other Key Game Theory Concepts

Threats and Rewards (Promises)

In game theory, players can achieve a strategic advantage through the *response rule*. A response rule sets one's action(s) as a response to another's action(s). Response rules are prevalent in our daily lives. The response rule can be defined in two ways: threats and rewards. Threats and promises are essentially the same; both are messages that players can send to each other to affect the other player in choosing a certain action. With a threat, failure to cooperate results in some type of negative payoff. With a reward or promise, cooperation results in some type of positive payoff. Both threats and rewards themselves can be defined further, generally as compellent or deterrent. A

compellent threat is meant to induce action from another, while a deterrent threat is meant to prevent future action from another.

Credibility

The *credibility* of the threat or promise is very important. If the threat or promise looks fundamentally unrealistic, then the threat or promise is not credible.

The Threat as a Strategy

Threats and rewards are strategic moves. Threats and rewards must be credible to influence the behavior of others. For this reason, smart players often display a pattern of fulfilling threats and promises throughout the game. When a player needs to offer a reward, he should not promise more than necessary to influence behavior. Likewise, threats should not be too big. Threats and promises of disproportionate scale can undermine the reputation of a player.

Games of Chance: Uncertainty and Risk

In game theory, chance and uncertainty are very important concepts. In some games, nature determines if one player is a winner or loser, and how much he wins. These are games of chance, such as rolling a pair of dice. Games of chance might be oneplayer games. In these games, nature affects the payoffs of the player based on his choices. Games of chance involve either risk or uncertainty, or both. In a game of chance involving risk, a player knows the probabilities of nature's response, such as tossing a coin. In a game of chance involving uncertainty, a player does not know the probabilities of nature's response.

Some games involve both chance and strategy, such as backgammon. Note that even in pure chance games, by randomizing a player can develop a strategy. Uncertainty is not only brought by nature, but also other players in the game. For instance, if a player randomizes his strategies in a game, other players are left with uncertainties regarding their opponents' moves.

One-player games against nature are not the subject of this book. Since our goal is to analyze strategic behavior and strategy, we concentrate on interactive decision-making environments, which involve two or more players.